Parametric Design Analysis of Magnetic Sensor Based on Model Order Reduction and Reliability-based Design Optimization

Sarbajit Paul[1], Arvind Rajan[2], Junghwan Chang[1], Ye Chow Kuang[2], Melanie Po-Leen Ooi[3]

[1]Mechatronics System Research Laboratory, Electrical Engineering, Dong-A University, Busan, 604-714, Korea, cjhwan@dau.ac.kr
[2]Advanced Engineering Platform, Monash University, 47500 Bandar Sunway, Malaysia, kuang.ye.chow@monash.edu
[3]School of Engineering & Physical Sciences, Heriot-Watt University, 62200 Putrajaya, Malaysia, m.ooi@hw.ac.uk

This paper presents a novel parametric design analysis of the magnetic sensor module used to find mover positions in linear motors. First, the sensor module is designed using the computationally inexpensive proper orthogonal decomposition (POD)-based model order reduction (MOR), coupled with a multiparameter moment matching method. Next, by modeling the surface response of the reduced model using a second-order polynomial, a reliability-based design optimization approach that considers the manufacturing uncertainties is employed to find the optimum sensor design. The results show that the proposed MOR technique is accurate and the reliability analysis performed during the design optimization can help reduce up to 40% of the excessive non-compliance costs for the manufacturers.

Index Terms—**Magnetic sensor, parametric model order reduction, optimization, probabilistic analysis.**

I. INTRODUCTION

IN THE design of magnetic sensors, computerized simulations such as the *finite element method* (FEM) followed by a design optimization procedure are widely used in recent years to achieve the optimal design parameters that satisfy the system constraints. However, for high-dimensional systems, the employment of FEM analysis can be computationally expensive. Furthermore, the parameter variations due to manufacturing uncertainties, which generally come from manufacturing processes and material properties, are often not taken into account during the optimization process. Thus, a large amount of defective (or non-compliant) products may be produced during the mass production stage, which could incur additional non-compliance costs for the manufacturers.

Addressing these shortcomings, the paper studies the use of the *parametric model order reduction* (PMOR) method and a probabilistic design optimization method, known as the *reliability-based design optimization* (RBDO), to design the magnetic sensors. For a simultaneously accurate and efficient analysis, the paper proposes 1) a *proper orthogonal decomposition* (POD), coupled with *multiparameter moment matching method* to generate a computationally cost effective reduced system model followed by 2) a novel response surface moment-based RBDO method. The sensor design obtained from the proposed method is then compared against that of the non-probabilistic optimization method in the context of design non-compliance (or failure) probability. The results show that the proposed method is accurate and that up to approximately 40% cost savings can be made if the system uncertainties are taken into consideration when optimizing the sensor design.

II. PROPOSED METHOD

A. Parametric study of based on model order reduction

Prior to carrying out the optimization process, the information of the design parameters is required. The behavior of the parameter variation is modeled using the multiparameter moment matching method-based model order reduction (MOR).

The overall computational operation can be divided into 1) initial parameter model based on FEM, 2) parameterization using the Taylor series, and 3) POD-based MOR. In FEM, the behavior of the electromagnetic system is studied using magnetic vector potential. Using the Galerkin method, the expression for potential can be written as

$$M(p_1, p_2, ..., p_n)U = F \qquad (1)$$

where, M_{mxm} is the stiffness matrix, U_{mx1} is the vector of the nodal potentials with m *degree of freedom* (DoF), F_{mx1} is the source vector and p_i ($i = \{1,2,...,n\}$) are the parameters. Using the Taylor series expansion, M in (1) can be expressed as [1],

$$M(p)U = \left(M_0 + \sum_{i=1}^{q} \Delta p_i M_i \right) U = F \qquad (2)$$

where, M_0 is the stiffness matrix with the initial parameters, Δp_i is the difference between initial and new parameter value, and M_i are coefficient matrices considering parameter variation.

To reduce the computational time required to solve the system (2), POD method is applied [2] and the reduced model can be written as

$$M_r(p)U_r = \left(\Psi^T M_0 \Psi + \sum_{i=1}^{q} \Delta p_i (\Psi^T M_i \Psi) \right) U_r = F_r \qquad (3)$$

where, Ψ is the discrete projection operator, calculated by the method of snapshot. Also, $M_r = \Psi^T M \Psi$, $F_r = \Psi^T F$. The DoF of the reduced model (3) is r ($r << m$).

B. Reliability-based design optimization: response surface moment-based reliability analysis

The parametric model data from the reduced model (3) is then used as the input to the *response surface methodology* (RSM) for optimization. The RSM is modeled using

$$G(\mathbf{X}) = a_{00} + \sum_{i=1}^{N} a_{0i} X_i + \sum_{i=1}^{N} a_{ii} X_i^2 + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} a_{ij} X_i X_j \qquad (4)$$

where $\{a_{00}, a_{0i}, a_{ii}, a_{ij}\}$ is the set of coefficients of the model; N is the number of variables, and $\mathbf{X}$ is the vector of variables.

In this paper, a new moment-based RBDO method is used. Here, the first four moments of G, $E[G^i]$ for $i = \{1,2,3,4\}$, are analytically calculated using Mellin transform [3]. The four

moments are then used to calculate the mean, standard deviation, skewness and kurtosis of G. The obtained parameters are then used by Pearson distribution fitting technique to model the output distribution of G and its reliability is directly inferred from the fitted distribution during the optimization procedure.

III. APPLICATION AND RESULTS

A magnetic sensor module, shown in Fig 1. is optimized using the proposed methods in Section II. The sensor module is used to find the mover position in the linear motor. It consists of a PM and three iron cores (I, II and III) respectively. A hall IC is placed between I and II, at the center point to detect the flux density B_x, along the X axis, when the sensor moves over the stator. The magnetization direction of PM is along the Y axis. For sensor output that is more robust to noise, the *total harmonic distortion* (THD) at center point is desired to be lesser than 4.5%, and it is the objective function to be minimized. Simultaneously, the flux density at the center point must be high enough to be detected by the Hall IC, and thus the center point *peak flux density* (PFD) higher than 0.16 T is desired. These are considered as the two design constraint in the optimization.

To carry out the optimization procedure, model parameters w and h_i, as shown in Fig. 1, were considered. The initial design specifications are taken as $(g_1, g_2, w, h_i, t) = (1, 2, 7, 3, 1)$. Using the initial design condition, FEM analysis of the sensor module was done. Then, w and h_i were varied to 6 mm and 2 mm respectively and FEM was done. Under these two cases, B_x variation was computed for one pole pitch movement of the mover. Using these data, the Taylor series expansion (2) was generated for parameterization. Then, a reduced model was achieved using (3). The process of parameterization and MOR were carried out in MATLAB platform. Once the reduced model is generated, the computation of B_x with respect to a new value of w and h_i can be very efficiently done by the reduced model without repeated the full high-dimensional FEM analysis.

The reduced model data was then used to obtain the RSM model (4) for PFD and THD through least squares method. The comparison of the surface plots obtained from the traditional FEM model and the polynomial RSM model is shown in Fig. 2. It shows that the proposed PMOR-based polynomial RSM model agrees with the traditional FEM model. Next, the following two design optimization scenarios were considered: 1) the non-probabilistic optimization which does not account for the manufacturing tolerance of w and h_i, and its effects on PFD and THD; and 2) RBDO (or probabilistic optimization) proposed in Section II-B, whereby the tolerance effects of w and h_i are considered during the optimization, with the failure probability of complying with the above mentioned constraints set to < 1%. Then, from the optimal designs obtained in both scenarios, the actual failure probabilities in adhering to the constraints were calculated using Monte Carlo simulation with 10^6 samples. The results are presented in Table I.

The optimal design attained from the non-probabilistic optimization adheres to the set constraints. Furthermore, the THD is the lowest among all the designs, promising a high-quality sensor design that is highly robust to noise. However, as the effects of manufacturing tolerance were not taken into

account during the optimization, the chances of failure in meeting the PFD constraint could go up to approximately 40%. This, in turn, brings additional non-compliance costs when remanufacturing or modifying the defective sensors. On the other hand, this can be avoided if RBDO is used in finding the optimal sensor design. The marginally higher THD (in Table I) would be the trade-off in avoiding the excessive expenses incurred due to non-compliance, making the probabilistic optimization tool highly valuable for the manufacturers.

In the future extended manuscript, all sensor parameters, material properties and their tolerance effects on the sensor design will be studied extensively with a more accurate RSM.
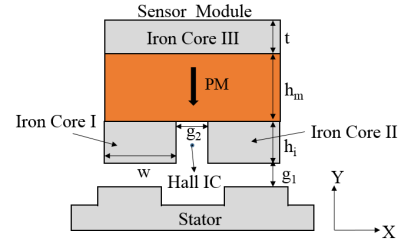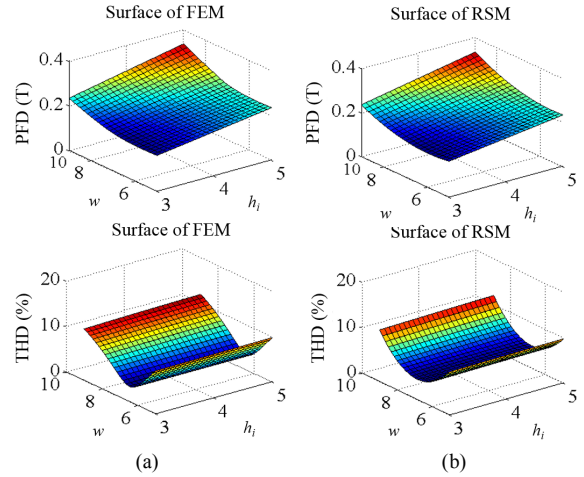

Fig. 1. Magnetic sensor module.

Fig. 2. PFD and THD surface responses of the (a) FEM model and (b) PMOR-based polynomial RSM model.

TABLE I
OPTIMAL DESIGN OF THE MAGNETIC SENSOR AND CONSTRAINT COMPLIANCE FAILURE PROBABILITIES

Optimization methods	Tolerance (mm)	Optimal design				Failure probability (%)	
		w (mm)	h_i (mm)	PFD (T)	THD (%)	PFD	THD
Traditional (Scenario 1)	± 0.1	6.45	3.00	0.16	3.25	41.71	0.00
	± 0.05					33.88	0.00
RBDO (Scenario 2)	± 0.1	6.80	3.15	0.17	4.42	0.01	0.04
	± 0.05	6.85	3.05	0.16	4.40	0.00	0.01

REFERENCES

[1] E. J. Yoo, *Parametric Model Order Reduction for Structural Analysis and control,* Ph.D. dissertation, Technische Universitat Munchen, 2010,.

[2] A. Chatterjee, "An Introduction to the proper orthogonal decomposition," *Current Science*, vol. 78, No. 7, pp. 808-817, 2000.

[3] Y. C. Kuang, A. Rajan, M. P.-L. Ooi, and T. C. Ong, "Standard uncertainty evaluation of multivariate polynomial," *Measurement,* vol. 58, pp. 483-494, 2014.